Two Days on PDEs

Bruno Pini Centenary Conference

Program and Abstracts

Accademia delle Scienze dell'Istituto di Bologna

June 21-22, 2018

https://events.unibo.it/bruno-pini-centenary-conference-bologna-2018

Contents

Program

June 21

10:00
Opening

10:15 - 10:55
E. Lanconelli
Caloric Harnack Inequality, Mean Value Theorem and Capacity: the Bruno Pini Work Towards Modern Parabolic Potential Theory

11:00 - 11:50
I. Capuzzo Dolcetta
Recent results about the maximum principle on unbounded domains

12:00 - 12:50
M. Bertsch
Measure-valued and discontinuous solutions of some evolution equations

15:00 - 15:50
A. Lunardi
Sobolev and BV functions in infinite dimension

16:00 - 16:50
J. Goldstein
The Agmon-Douglis-Nirenberg Problem for Dynamic Boundary Conditions

17:00 - 17:50
S. Terracini
Regularity of the optimal sets for spectral functionals and the free boundary for the vectorial Bernoulli problem

June 22

09:30 - 10:20

M. A. Herrero
Evolutionary Politics: lessons from the immune system

10:30 - 11:20

G. Ruiz Goldstein
New Results on Instantaneous Blowup in $\mathbb{H}^n$

11:30 - 12:20

U. Bottazzini
On Harnack's contributions to potential theory

14:30 - 15:20

V. Barbu
Mild solutions to a second order Hamilton- Jacobi equation arising in mathematical finance

15:30 - 16:20

S. Salsa
Regularity in free boundary problems with distributed sources

Abstracts

Viorel Barbu
Academia Română - Filiala Iași

Mild solutions to a second order Hamilton- Jacobi equation arising in mathematical finance.

Existence and uniqueness of a mild solution to the dynamic programming equation corresponding to optimal control associated with the Heston stochastic volatility control is studied. The approach is based on nonlinear semigroup theory in the space L^1.

Michiel Bertsch
Università degli Studi di Roma "Tor Vergata"

Measure-valued and discontinuous solutions of some evolution equations.

Motivated by an application to stratified turbulent flows in oceanography, I shall discuss some singular solutions of nonlinear PDE's of evolutionary type, in particular discontinuous and Radon measure valued solutions. We focus on different regularizations of backward parabolic equations, first order scalar conservation laws, and, if time permits, a toy problem for a Hamilton-Jacobi equation. Most of this lecture is based on collaborations with L. Giacomelli, F. Smarrazzo, A. Terracina and A. Tesei.

Umberto Bottazzini
Università degli Studi di Milano

On Harnack's contributions to potential theory.

One of Bruno Pini's most quoted papers deals with his (and Hadamard's) contemporary derivation of a theorem analogue to a theorem by Harnack on harmonic functions. In the talk I will focus on Harnack's contributions to potential theory and his solution to Dirichlet problem as presented in his 1887 book as well as to Kellog's 1929 book on potential theory to which Pini himself referred in his paper.

Italo Capuzzo Dolcetta
Sapienza Università di Roma and Gnampa-Indam

Recent results about the maximum principle on unbounded domains.

I will give an overview of some results concerning the validity of sign propagation property

$$u0 \text{ on } \partial\Omega, \qquad F(x,u,Du,D^2u)0 \text{ in } \Omega \text{ implies } u \leq 0 \text{ in } \Omega$$

in an unbounded domain Ω satisfying either measure-type or geometric conditions related to the directions of ellipticity of the (possibly) degenerate fully nonlinear mapping F.

Jerome Goldstein
The University of Memphis

The Agmon-Douglis-Nirenberg Problem for Dynamic Boundary Conditions.

Of concern are certain reaction-diffusion systems with total mass bounded in the L_1 norm. The solution of this problem requires new results from the study of a linear uniformly parabolic heat equation on a bounded domain with dynamic (or Wentzell) boundary conditions incorporating the Laplace-Beltrami operator. We prove that the semigroup governing this problem is analytic in the right half

plane in L_p for all $p > 1$ and for C in the supremum norm. The proof is long and delicate. This is joint work with Gisele Ruiz Goldstein and Michel Pierre.

Miguel A. Herrero
Real Academia de Ciencias and Universidad Complutense de Madrid

Evolutionary Politics: lessons from the immune system.

Consider a society consisting of a large number of individuals (about 10 13, several orders of magnitude larger than today´s world population) organized in hundreds of different social groups (current count of UN countries being about 200) and employed in many different jobs. Assume further a huge immigration rate (about 1011 new arrivals per day) coupled to a high mortality rate, so as to balance the previous figure. Such society enjoys full employment, and wealth is shared by all individuals. Order is maintained by an extremely efficient police force that keeps threatening aliens at bay. What kind of government could possibly be up to the task of ruling such society?
The answer is simple: none. Anarchy prevails in the society we have summarily described, which is not located at the remote island of Utopia: it is just you (or me) and no central organ of control is in charge of its operation. Its efficient performance is an emergent property, resulting from individual decisions of its cells, and is not centrally regulated from any commanding headquarters. We shall describe in this lecture some features of one of the cornerstones of this complex structure, the immune system, and will shortly remark on other cell regulation properties which show a distinct emergent character as well.

Ermanno Lanconelli
Alma Mater Studiorum Università di Bologna

Caloric Harnack Inequality, Mean Value Theorem and Capacity: the Bruno Pini Work Towards Modern Parabolic Potential Theory.

We describe the pioneering work of Bruno Pini towards the modern Potential Analysis of linear second order parabolic Partial Differential Equations. We mainly focus on the caloric Harnack Inequality discovered by Bruno Pini in 1954, jointly, and independently, with Jacques Hadamard. Pini made of this inequality the crucial tools in his construction of a Wiener-type solution to the "Dirichlet problem" for the Heat equation. To this end he also introduced an average operator on the level set of the Heat kernel, characterizing caloric and sub-caloric

functions, in analogy with the classical Gauss-Koebe, Blaschke-Privaloff and Saks Theorems for harmonic and sub-harmonic functions. Pini also established, and used, the notion of caloric capacity to study the boundary behavior of his Wiener-type solution to the first boundary value problem for the Heat equation.

Alessandra Lunardi
Università degli Studi di Parma

Sobolev and BV functions in infinite dimension.

In Hilbert or Banach spaces X endowed with a good probability measure μ there are a few "natural" definitions of Sobolev spaces and of spaces of bounded variation functions. The available theory deals mainly with Gaussian measures and Sobolev and BV functions defined in the whole X, while the study and Sobolev and BV spaces in domains, and/or with respect to non Gaussian measures, is largely to be developed. As in finite dimension, Sobolev and BV functions are tools for the study of different problems, in particular for PDEs with infinitely many variables, arising in mathematical physics in the modeling of systems with an infinite number of degrees of freedom, and in stochastic PDEs through Kolmogorov equations. In this talk I will describe some of the main features and open problems concerning such function spaces.

Gisele Ruiz Goldstein
The University of Memphis

New Results on Instantaneous Blowup in $\mathbb{H}^n$***.***

Consider the heat equation

$$u_t = u + V(x)u$$

for $xin\mathbb{R}^N$ with a positive potential $V(x)$: If V is "too singular", then this equation may not have any positive solutions, as was discovered in 1984. We shall discuss the history of the problem as well as later developments, including new results obtained in 2017. The Euclidean space $\mathbb{R}^N$ can be replaced by the Heisenberg group $\mathbb{H}^n$ and other Carnot groups, and the heat equation can be relaced by the Ornstein-Uhlenbeck equation and other related equations. Some nonlinear results will be mentioned. Scaling plays a critical role.

Sandro Salsa
Politecnico di Milano

Regularity in free boundary problems with distributed sources.

We describe recent results obtained in collaboration with Daniela De Silva and Fausto Ferrari on two-phase free bounday problems in presence of distributed sources. The focus will be mainly on higher regularity and related open problems.

Susanna Terracini
Università degli Studi di Torino

Regularity of the optimal sets for spectral functionals and the free boundary for the vectorial Bernoulli problem.

In this talk we deal with the regularity of optimal sets for a shape optimization problem involving a combination of eigenvalues, under a fixed volume constraints. As a model problem, consider

$$\min\Big\{\lambda_1(\Omega)+\cdots+\lambda_k(\Omega)\ :\ \Omega\subset\mathbb{R}^d,\ \text{open}\ ,\ |\Omega|=1\Big\},$$

where $\lambda_i(\cdot)$ denotes the eigenvalues of the Dirichlet Laplacian and $|\cdot|$ the d-dimensional Lebesgue measure. We prove that any minimizer Ω_{opt} has a regular part of the topological boundary which is relatively open and $C^{1,\alpha}$ regular and that the singular part has Hausdorff dimension smaller than $d-d^*$, where $d^*\geq 3$ is the minimal dimension allowing the existence of minimal conic solutions to the bow-up problem.
We examine the link between this and the problem of regularity of the free boundary for a vector-valued Bernoulli problem, with no sign assumptions on the boundary data. More precisely, given an open, smooth set of finite measure $D\subset\mathbb{R}^d$, $\Lambda>0$ and $\varphi_i\in H^{1/2}(\partial D)$, we deal with

$$\min\left\{\sum_{i=1}^k\int_D|\nabla v_i|^2+\Lambda\Big|\bigcup_{i=1}^k\{v_i\neq 0\}\Big|\ :\ v_i=\varphi_i\ \text{on}\ \partial D\right\}.$$

We prove that, for any optimal vector $U=(u_1,\dots,u_k)$, the free boundary $\partial(\cup_{i=1}^k\{u_i\neq 0\})\cap D$ is made of a regular part, which is relatively open and locally the graph of a C^∞ function, a (one-phase) singular part, of Hausdorff dimension at most $d-d^*$, for a $d^*\in\{5,6,7\}$, and by a set of branching (two-phase) points, which is relatively closed and of finite $\mathcal{H}^{d-1}$ measure having a stratified

structure itself.
These are joint works with Dario Mazzoleni and Bozhidar Velichkov.

References

[1] D. Mazzoleni, S. Terracini and B. Velichkov, Regularity of the optimal sets for spectral functionals, Geom. Funct. Anal. 27 (2017), no. 2, 373-426

[2] Dario Mazzoleni, Susanna Terracini, Bozhidar Velichkov, Regularity of the free boundary for the vectorial Bernoulli problem, preprint 2018